Thermodynamics of Gas-Solid Flow

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New thermodynamic fundamentals for vertical pneumatic conveying are proposed with derivation based strictly on mass conservation and continuum concept. For a 1-D, vertical, steady gas-solid flow, these new mass and energy balances allowed the derivation of a new nondimensional energy factor containing the pressure drop. This energy factor correlated quite well with the difference between the inlet gas velocity and solids terminal velocity, when tested against high-pressure, 15-m-high, vertical pneumatic transport data from Institute of Gas Technology. The energy factor was also checked against atmospheric-pressure data of the 5-m lift line of the Pennsylvania State University. The new energy factor covers quite well both the atmospheric- and high-pressure sets of data (923 tests) including two heights, lean- and dense-phase transports, several pipe diameters, broad and narrow particle-size distributions, and different materials. The dissipation terms in gas-solid flow were also clearly identified.

Introduction

Although the simplified case of uniform (averaged) gas velocity, particle velocity, and distribution over the cross-sectional of a pipe in steady gas-solid flow could be considered quite a simple problem, the state of the art, as applied to pneumatic conveying, still leaves much to be desired (Knowlton, 1986).

The classical hydrodynamic models for one-dimensional (1-D) steady gas-solid flow state continuity and momemtum equations for each of the phases obtain a mixture momemtum equation by the simple addition of the gas and solids momemtum ones. From a strict point of view, following Teo and Leung (1984), Arastoopour and Gidaspow (1979a,b) and Arastoopour et al. (1982), we usually have three nonlinear first-order differential equations which describe the system, that is, gas continuity, solids continuity, and mixture momemtum conservation. Also, we want to determine four continuous functions: the gas and solids velocities, the gas (or solid) void fraction, and the gas pressure. Therefore, we need one more differential equation. The variations of gas density are accounted for by means of its thermodynamic relationship with pressure and temperature through an equation of state (generally ideal gas), being usual to assume isothermal flow (Rudinger, 1980; Arastoopour and Gidaspow, 1979a,b; and Arastoopour et al., 1982). These three differential equations are the noncontroversial and widely admitted ones (Teo and Leung, 1984). Although the fourth equation, which would close the above set of differential equations, still remains an open question.

Arastoopour and Gidaspow (1979a,b) and Arastoopour et al. (1982) exhaustively discuss their own proposals for the fourth equation and those of the specialized literature comparing the analytical results of a total of four models with experimental measurements. It is clearly explained in their conclusions that comparison of any hydrodynamic model to experimental data involves the fitting of an inlet solid void fraction.

Although these hydrodynamic models (Arastoopour and Gidaspow, 1979a,b) seem to predict well the pressure drop with narrow particle-size distributions, some checks with broad particle-size distribution indicated the need for better estimates of particle interaction forces and voidage to relate accurately the main variables of the system (Arastoopour et al., 1982).

By the other side, the empirical correlations (see, for example, the exhaustive revisions about vertical pneumatic conveying pressure drop correlations of Leung and Wiles (1976), Teo and Leung (1984) and Knowlton (1986)) work with average values (for all the riser) of the velocities and void fractions of the solid and gas, avoiding the needing of the controversial fourth equation.

These correlations hand a simple equation for the total pressure drop, which is the sum of the pressure drop contri-

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butions due to acceleration, gravity, and friction. The friction term is the more studied (see the revision of Leung and Wiles (1976)) being usual to work with different correlations for near atmospheric conditions and high-pressure systems. Also, there is some confusion about the acceleration term: in many cases, the recommended correlations for predicting pressure gradient (Konno and Saito, 1969; Capes and Nakamura, 1973; Nakamura and Capes, 1973; Yang, 1978) were only written for data taken in the fully developed region. Also, the extent to which the flow is "developed" was measured by reading the pressure drops at several locations, where the slope of the pressure gradient was constant.

However, as pointed out by Fan and Zhu (1998), the pressure drop over the developing regions of gas-solid flows (where the particles are accelerated) is often significant or even dominant, with typical developing regions the particle entrances and regions immediately beyond pipe bends.

In his extensive review, Knowlton (1986) concludes that the pressure drop correlations were evaluated for a very limited amount of data, the best correlations for pneumatic transport can generally be expected to predict parameters no more accurately than 30 to 40%, and, if the correlations were to be applied to high temperature and/or high-pressure systems, their accuracy could be much worse than 30 to 40%, since nearly all the correlations were developed from data obtained at ambient conditions. Also, Teo and Leung (1984) affirm that, although there are many correlations for predicting pressure drop in dilute phase conveying, there is considerable doubt whether the correlations may be extrapolated with confidence, if at all.

Unfortunately, it seems that neither the hydrodynamical models nor the empirical correlations have given satisfactory results for the prediction of the pressure drop, probably the most fundamental parameter for design and operation purposes (Arastoopour and Gidaspow, 1979a).

Recently, a new approach to the gas-solid modeling has been proposed by the author (Collado and Muñoz, 1997). The main idea is quite simple: if the two phases have different velocities, they cannot cover the same distance (the length of control volume) in the same time. Thus, the time scale of the two phases cannot be the same, and we should scale time-dependent magnitudes of one phase to the other one before combining them. Indeed, the scale factor is the slip ratio or gas-solid velocity ratio. Based on this new thermodynamic point of view, new mass, energy, pressure-energy, and entropy equations for vertical pneumatic conveying have been recently derived by the author, allowing to derive a new nondimensional energy factor which contains the total pressure drop.

The energy factor correlated quite well with a solids reference velocity, namely, the difference between the inlet superficial gas velocity and the solids terminal velocity. It was tested against high-pressure, 15 m height, vertical pneumatic transport data (Knowlton and Bachovchin, 1976) from the Institute of Gas Technology.

In this article, after a brief review of this new approach based on thermodynamic fundamentals, the comparison of the energy factor against atmospheric-pressure data taken in the 5.8 m lift line of the Pennsylvania State University (Jones et al., 1966, 1967) is analyzed, and a strong dependence of the energy factor on the solids reference velocity has been also verified for the atmospheric-pressure data. A brief outline of this analysis was already sketched in Collado (2000d).

As a major result, a general correlation of the energy factor has been found, which is able to cover quite well both the atmospheric- and the high-pressure set of data (923 tests). It includes two lift line heights, lean and dense phase transport, several pipe diameters, broad and narrow particle-size distributions, and several quite different materials. The pressure drop for both the atmospheric- and the high-pressure data was taken just from the solid inlet, so including the acceleration region.

Furthermore, following the new point of view, new global momemtum and mechanical energy balances for a gas-solid flow are presented. The comparison of the mechanical energy balance with the pressure-energy equation allows to identify the mechanical losses in a gas-solid flow, which are logically due to the velocity gradients near the wall and the slip between the phases.

Comment that the same expressions of these one-dimensional steady two-phase flow balances have been already compared successfully with flow boiling data (Collado, 2000a,b,c). These balances are merely particular cases of new general multiphase flow fundamentals based on mass conservation and the continuum concept, and presented elsewhere (Collado, 1999).

New Mass and Energy Balances for 1-D Steady Gas-Solid Flow

New mass balance

The gas and solid continuity equations of the new model are exactly the same as the classical ones for a 1-D, nonreacting, steady, gas-solid flow

$$\dot{m}_g = \rho_g v_g \epsilon A_c = \text{constant} \Rightarrow \frac{d}{dz}\left(\rho_g v_g \epsilon A_c \right) = 0 \tag{1}$$

$$\dot{m}_s = \rho_s v_s (1-\epsilon) A_c = \text{constant} \Rightarrow \frac{d}{dz}\left[\rho_s v_s (1-\epsilon) A_c \right] = 0 \tag{2}$$

where v, $\dot{m}$, ρ, and A_c are velocity, mass-flow rate, material density, and the cross-sectional area of the line, respectively; subscripts g and s stand for gas and solid, respectively. Finally, ϵ is the porosity or gas void fraction and z is the vertical coordinate.

The main novelty in the proposed mass conservation set of equations (Collado and Muñoz, 1997) was the inclusion of a new equation which stated that, at steady state, the gas-solid velocity ratio (in the following slip ratio and denoted by S) is a constant throughout the duct

$$\frac{d}{dz}\left(\frac{v_g}{v_s} \right) = \frac{d}{dz}(S) = 0 \Rightarrow S = \frac{v_{gi}}{v_{si}} = \frac{v_{go}}{v_{so}} = \text{constant} \tag{3}$$

This assertion was exclusively based on mass conservation and the continuum concept, and it was practically derived through the integration of the mass contained in the duct, previously divided into differential control volumes of variable length proportional to the gas velocity, that is, following an Eulerian-Lagrangian approach. Equation 3 would be the new proposed fourth equation which would close the system of differential equations discussed by Teo and Leung (1984).

Equation 3 also allows to justify that either of the two phase velocities (gas or solids) could be the representative velocity of the whole mixture. With that being the case, we first state the classical mixture density, ρ_m

$$\rho_m = \rho_g \epsilon + \rho_s(1-\epsilon) \quad (4)$$

Then, combining Eqs. 1 and 3, we can yield two global mass conservation equations, both valid, with the only difference being the time scale used. In the first one, the gas velocity multiplies the mixture density, suggesting that it would be the representative velocity of the mixture

$$\frac{d}{dz}\left[\dot{m}_g + \left(\frac{v_g}{v_s}\right)\dot{m}_s\right] = 0 = \frac{d}{dz}\left[\rho_g v_g \epsilon A_c + \rho_s v_g (1-\epsilon) A_c\right] = \frac{d}{dz}\left[\rho_m v_g A_c\right] = 0 \quad (5)$$

In the second one, the solid velocity acts as the global velocity of the mixture

$$\frac{d}{dz}\left[\left(\frac{v_s}{v_g}\right)\dot{m}_g + \dot{m}_s\right] = 0 = \frac{d}{dz}\left[\rho_g v_s \epsilon A_c + \rho_s v_s (1-\epsilon) A_c\right] = \frac{d}{dz}\left[\rho_m v_s A_c\right] = 0 \quad (6)$$

New energy balance

Following Rudinger (1980), the specific energy of the mixture e_m (in Joules per unit mass of mixture) is defined as

$$e_m = \left[\rho_s(1-\epsilon)/\rho_m\right] e_s + \left(\rho_g \epsilon/\rho_m\right) e_g = \left[\rho_s(1-\epsilon)/\rho_m\right] \times \left(u_s + v_s^2/2 + gz\right) + \left(\rho_g \epsilon/\rho_m\right)\left(u_g + v_g^2/2 + gz\right) \quad (7)$$

Now, taking the gas velocity as the representative velocity of the mixture, we can derive a new expression of the global energy balance (in Watts) for a vertical, steady state, two-phase flow between two stations i (inlet), o (outlet)

$$(\rho_m e_m)_o v_{go} A_c - (\rho_m e_m)_i v_{gi} A_c = \dot{Q}_m - \dot{W}_m \quad (8)$$

where Q_m and W_m are the heat and the work exchanged (per unit time) by the gas-solid mixture as a whole, respectively; and u and gz are the internal energy and the gravity terms, respectively.

Substituting Eq. 7 in Eq. 8, one obtains

$$\left(u_{so} + v_{so}^2/2 + gz_o\right)\rho_s(1-\epsilon_o) v_{go} A_c + \left(u_{go} + v_{go}^2/2 + gz_o\right) \times \rho_{go}\epsilon_o v_{go} A_c - \left(u_{si} + v_{si}^2/2 + gz_i\right)\rho_s(1-\epsilon_i) v_{gi} A_c - \left(u_{gi} + v_{gi}^2/2 + gz_i\right)\rho_{gi}\epsilon_i v_{gi} A_c = \dot{Q}_m - \dot{W}_m \quad (9)$$

In this equation, the pressure acting over the entire cross section of the duct is the only force which can produce work (Rudinger, 1980). Then $\dot{W}_m$ will correspond with the product of the force (pressure) by the displacement per unit time (velocity) of the section where it is applied. However, working with the whole system (gas + solid), we would have to use, by coherence, the representative velocity of the global gas-solid system, which is the gas velocity as we have chosen above

$$\dot{W}_m = P_o v_{go} A_c - P_i v_{gi} A_c \quad (10)$$

Substituting Eq. 10 in Eq. 9, while taking into account the gas and solid mass-flow rates expressions (Eqs. 1 and 2) and the slip ratio constancy (Eq. 3), and after grouping separately the gas and solid terms, we yield (see more details in Collado and Muñoz (1997))

$$\left\{\left(u_{go} + P_o/\rho_{go}\right) - \left(u_{gi} + P_i/\rho_{gi}\right) + \left(v_{go}^2/2 - v_{gi}^2/2\right) + gH\right\}\dot{m}_g + \left\{\left(u_{so} - u_{si}\right) + \left(v_{so}^2/2 - v_{si}^2/2\right) + gH + \left(P_o - P_i\right)/\rho_s\right\}\left(v_g/v_s\right)\dot{m}_s = \dot{Q}_m \quad (11)$$

where H is the vertical distance from the inlet to the outlet $H = z_o - z_i$.

We now apply the classical thermodynamic definition of gas enthalpy as a combination of gas internal energy, gas pressure, and gas density (Kestin, 1979). For the solids (an incompressible substance), we use the expression of internal energy in the function of the specific heat (Kestin, 1979), assuming this is constant (Rudinger, 1980).

Finally, dividing Eq. 11 by the gas mass-flow rate $\dot{m}_g$, we state the mixture energy equation in Joules per unit mass of gas

$$\left\{\left(h_o - h_i\right) + \left(v_{go}^2 - v_{gi}^2\right)/2 + gH\right\} + \left\{c\left(T_{so} - T_{si}\right) + \left(v_{so}^2 - v_{si}^2\right)/2 + gH - \Delta P/\rho_s\right\}\cdot S \cdot \eta = q \quad (12)$$

where q is the total heat absorbed by the mixture per unit mass of gas, h is the gas enthalpy, c is the specific heat of solid material, T_s is the solid temperature (Kelvin), ΔP is the pressure drop between the inlet and the outlet of the lift line, and $\Delta P = P_i - P_o$, and $\eta = \dot{m}_s/\dot{m}_g$ is the load ratio or solid-gas mass flow ratio, constant through the duct.

Pressure-Energy Equation and the Entropy Balance For Gas-Solid Flow

In the energy balance of Eq. 12, it is more convenient to separate the thermal terms from the mechanical ones with the objective of identifying the mechanical irreversibilities or losses due to velocity gradients or friction (between the mixture and the wall, and between the phases). Classically, this has been done through the comparison (Zucker, 1977; Hall, 1956) of the momentum equation with the pressure-energy equation which is usually derived from the energy balance with the help of the second *Tds* equation (Bejan, 1988). For the gas

$$dh = dP/\rho_g + T_g ds_g \Rightarrow h_o - h_i = \int dP/\rho_g + \int T_g ds_g \quad (13)$$

where dP/ρ_s is the differential gas expansion work, T_g is the gas absolute temperature, and s_g is the gas entropy. Substituting Eq. 13 in Eq. 12 and reordering the terms

$$\begin{aligned} -\int dP/\rho_g - \left(v_{go}^2 - v_{gi}^2\right)/2 - gH \\ = \left\{-q + \eta\cdot S\cdot\left[c(T_{so} - T_{si})\right] + \int T_g ds_g\right\} \\ + \left[\left(v_{so}^2 - v_{si}^2\right)/2 + gH - \Delta P/\rho_s\right]\cdot\eta\cdot S \quad (14)\end{aligned}$$

The first term in brackets on the righthand side represents the lost work due to irreversibilities or entropy generation (Bejan, 1988) of the whole system in Joules per unit mass of gas

$$\begin{aligned} -q - \eta\cdot S\cdot\left[c(T_{so} - T_{si})\right] + \int T_g ds_g = -q + \eta\cdot S\cdot\int T_s ds_s \\ + \int T_g ds_g = w_{\text{lost}} \quad (15)\end{aligned}$$

Comment that, for the particles (an incompressible substance), the entropy differential is given by $ds_s = c\cdot dT_s/T_s$ (Kestin, 1979). Equation 15 would be the entropy balance of the global system (gas + solids).

Thus, the energy equation for the mixture (in Joules per unit mass of gas) becomes

$$\begin{aligned} -\int dP/\rho_g - \left(v_{go}^2 - v_{gi}^2\right)/2 - gH = w_{\text{lost}} \\ + \left[\left(v_{so}^2 - v_{si}^2\right)/2 + gH - \Delta P/\rho_s\right]\cdot\eta\cdot S \quad (16)\end{aligned}$$

Equation 16 would be the pressure-energy equation for a gas-solid flow, since it preserves the inherent properties of the energy equation and yet applies them to pressure variation (Hall, 1956; Zucker, 1977). This equation would be equivalent to the mixture momentum equation used by Arastoopour and Gidaspow (1979a,b). In conclusion, the hydrodynamic model proposed here would have the following four differential equations: gas continuity (Eq. 1), solid continuity (Eq. 2), slip ratio constancy (Eq. 3), and the mixture pressure-energy equation (Eq. 16).

Energy factor

From Eq. 16, it is clear that the second term on the lefthand side represents the useful or effective work of the gas on the solids (effective work) in Joules per unit mass of gas (Collado and Muñoz, 1997), which we denote by w_e

$$w_e = \left[\left(v_{so}^2/2 - v_{si}^2/2\right) + gH - \Delta P/\rho_s\right]\cdot\eta\cdot S \quad (17)$$

Further, by convenience, we define the effective work of the gas on the solids per unit mass of solid in the pipe (in short, solid effective work), denoted by w_{se}, as

$$w_{se} = \left[\left(v_{so}^2/2 - v_{si}^2/2\right) + gH - \Delta P/\rho_s\right] \quad (18)$$

In many vertical pneumatic conveying systems, as in the IGT and Pennsylvania University tests, it is quite usual that the vertical component of the solid velocity is near zero at the solids feeding port. So, in these cases, $v_{si} = 0$, and the coherent expression of the effective work and the solid effective work would be

$$w_e = \left[v_{so}^2/2 + gH - \Delta P/\rho_s\right]\cdot\eta\cdot S \quad (19)$$

and

$$w_{se} = \left[v_{so}^2/2 + gH - \Delta P/\rho_s\right] \quad (20)$$

respectively.

Dividing the effective work by a reference kinetic energy of the solids, a nondimensional energy factor, denoted by f_e, was obtained

$$f_e = w_{se}/\left(v_{s-\text{ref}}^2/2\right) \quad (21)$$

where, based on the usual engineering procedures (Knowlton and Bachovchin, 1976), the solid reference velocity was defined (Collado and Muñoz, 1997) as the difference between the superficial inlet gas velocity U_{gi} and the terminal velocity in the solids v_t (two velocities which can be measured or calculated with ease)

$$v_{s-\text{ref}} = U_{gi} - v_t \quad (22)$$

Finally, highlight that the necessary outlet solid velocity v_{so} in Eqs. 19 and 20 was calculated with the help of Eq. 3 or the constancy of the slip ratio through the line. In these tests, as $v_{si} = 0$, we would have an indetermination in Eqs. 2 and 3. However, due to Eq. 3, we only need a "reasonable" value of the slip ratio in the line, which was approximated with the help of the above defined solid reference velocity

$$v_{go}/v_{so} = S = v_g/v_s \approx U_{gi}/v_{s-\text{ref}} = U_{gi}/\left(U_{gi} - v_t\right) \quad (23)$$

This approximation will be discussed later.

Energy Factor vs. Atmospheric-Pressure, Vertical Pneumatic Conveying Data

In Collado and Muñoz (1997) a comparison was made of the energy factor (Eq. 21) with experimental pressure drop data of the vertical (15.115 m), high-pressure, pneumatic transport of Montana lignite and siderite ore (until 46.9 bar of inlet pressure and 746.6 kg/m^{2}s of solids load) from the research facility of the Institute of Gas Technology (IGT) (Knowlton and Bachovchin, 1976). An excellent correlation between this energy factor, which includes the pressure drop, and the proposed reference solid velocity (Eq. 22), was obtained.

We are going to compare here the same effective work, solid effective work, and energy factor and the same calcula-

Table 1. Characteristics of Materials and Vertical Pneumatic Transport Tests of Knowlton and Bachovchin (1976) and Jones et al. (1966, 1967)

Solids	Density (kg/m^3) ρ_s	Surface D_p (μm)	Std. Dev. (μm)	Terminal Vel., v_t (m/s)	Pres. (bar) /Gas/ ID (mm)	Inlet Gas Vel. (m/s) U_{gi}	Solid-Gas Mass-Flow Ratio $\dot{m}_s/\dot{m}_g$	Solids Load ($kg/m^2 \cdot s$) $\dot{m}_s/A_c$
IGT (135 tests)								
Montana lignite	1,259	362	516	1.1–2.2 (Calc.)	4.8–46.9/N_2/73.6	2.5–24.1	0.3–5.9	77.1–203.5
Siderite ore	3,908.5	160	108	0.8–1.1 (Calc.)	10.8–32.1/N_2/73.6	2–7.8	1.3–14.1	195.2–746.6
Penn. Univ (788 tests)								
Glass beads No. 108	2,530.9	349	28.8	2.6 (Exp.)	Atmosph./Air/ 7.75, 10.21	5.8–33.5	0.3–13.3	11.7–115.5
Glass beads No. 106	2,498.9	578	45.8	4.1 (")	" 7.75, 10.21, 22.1	6.5–33.2	0.2–14.8	4.8–109.3
Fused Alumina No. 100	3,892.5	201	30.8	1.2 (")	" 7.75, 10.21	3–15.5	1.9–16.7	12.6–123.8
Fused Alumina No. 60	3,956.6	453	57.6	2.4 (")	" 7.75, 10.21	4–32.6	0.5–9.6	21.5–71.9
Fused Alumina No. 36	3,892.5	740	61	3.9 (")	" 7.75, 10.21, 22.1	6–32.3	0.2–9	3.4–67.9
Zircon silica No. 1	3,476	561	50.7	4.2 (")	" 7.75, 10.21	7.1–32.6	0.6–12.2	19.5–106.9
Zircon silica No. 2	3,331.8	382	40.4	2.8 (")	" 7.75, 10.21	5.1–32.6	0.6–11.5	8.7–104.5
Steel shot A	7,560.7	490	43	6.6 (")	" 7.75, 10.21	3.9–37.5	0.8–11.6	15.5–165.5
Steel shot C1	7,592.8	273	21.6	4.1 (")	" 7.75, 10.21	8.1–37.2	0.9–22.3	45.7–254.7
Steel shot C2	7,608.8	322	27.3	4.6 (")	" 7.75, 10.21	7.2–37.8	1.1–16.1	49.9–220
Steel shot D1	7,256.4	765	38.5	9.2 (")	" 7.76, 10.21	11.5–38.1	0.7–8.8	37.7–139.4
Steel shot D2	7,240.3	660	38.8	8.4 (")	" 10.21	3.2–37.8	0.7–7.6	38.6–125.8

tion procedure already proposed in Collado and Muñoz (1997) with experimental data for atmospheric-pressure, vertical pneumatic transport tests made by Jones et al. (1966, 1967) from the Pennsylvania State University.

The height of the line was 5.79 m, and, until twelve different materials were tested, all of them had a very narrow size distribution. Three different steel lift lines of 3/8 in., 1/2 in., and 1 in. O.D. were used. The authors also directly measured the terminal velocity of all the materials.

In Table 1 the main characteristics of the materials used in the pressure drop tests of the Penn State Univ. are shown. For comparison, the same data for the IGT (Knowlton and Bachovchin, 1976) tests have been also included. Due to the broad size distribution of the materials tested by IGT, their global terminal velocity was a weighted mean of the calculated individual terminal velocities of each size interval (Collado and Muñoz, 1997). The 923 tests of Table 1 include high-pressure (until 47 bar) and atmospheric pressure tests, two line heights (15.2 and 5.8 m), diluted and dense phase ($\eta > 10$) transports, four pipe diameters, broad and narrow particle-size distributions, and 14 quite different materials.

In Figure 1, the pressure drop vs. the measured inlet gas velocity for all the vertical pneumatic transport tests of Table 1 is shown, ΔP ranging from a few millibar to 1,200 milibar, and U_{gi} from 2 m/s to 38 m/s. Figure 2 and Figure 3 show the dependency of the effective work (Eq. 19) and the solid effective work (Eq. 20) on the superficial velocity U_{gi} both for the two set of data.

In Figure 4 the dependency of the energy factor (Eq. 21) on the solid reference velocity (Eq. 22) for the two set of data is presented in logarithmic scale. The energy factor correlation for the IGT data was formerly presented in Collado and Muñoz (1997) and some preliminary results of the comparison with the Pennsylvania set of data were outlined in Collado (2000d). Obviously, *it would be interesting to find a general correlation which covered all the data presented.*

We have found that substituting in the energy factor of the Penn data their lift line height (5.79 m) by the value of the IGT installation height (15.115 m) the atmospheric data cor-

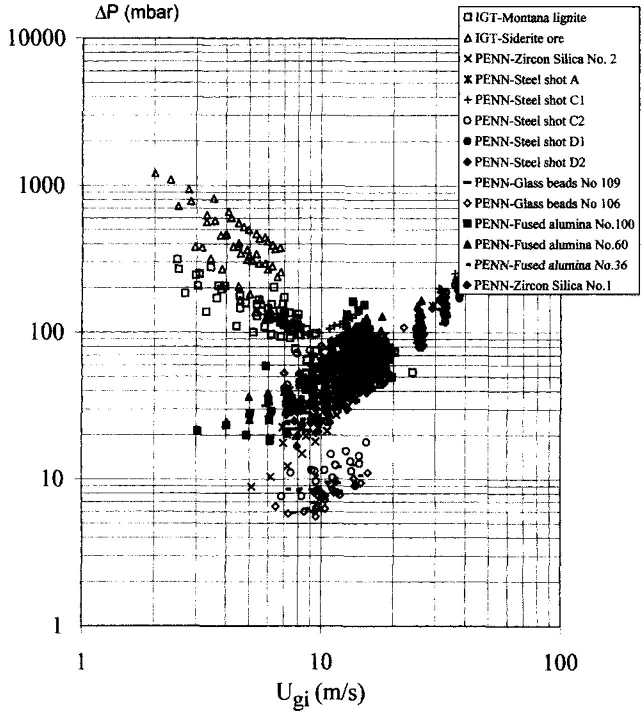

Figure 1. Pressure drop vs. inlet superficial gas velocity for IGT and Penn. Univ. tests.

relation coincides with the high-pressure one (Figure 5). The cubic polynomial shown in Figure 5 fits the 923 values quite well, the regression coefficient being 0.9992.

This general correlation of the energy factor shown in Figure 5 is based on the IGT height. Logically, if we would want to find the energy factor for the Penn tests with other height, we should merely undo the process

$$(f_e)_{\rm PENN} = (f_e)_{\rm IGT} - [g(H_{\rm IGT} - H_{\rm PENN})]/(v_{s-\rm ref}^2/2) \quad (24)$$

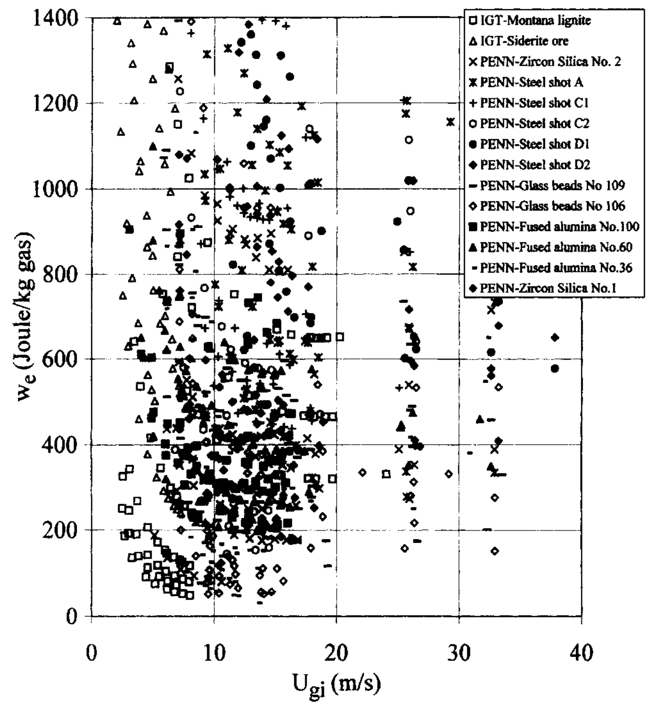

Figure 2. Effective work vs. inlet superficial gas velocity for IGT and Penn. Univ. data.

Of course, any possible extension of this procedure should be first carefully checked with more data from other heights.

Discussion

A key point of the new model is the slip ratio constancy along the line (Eq. 3), the new balances being based on it.

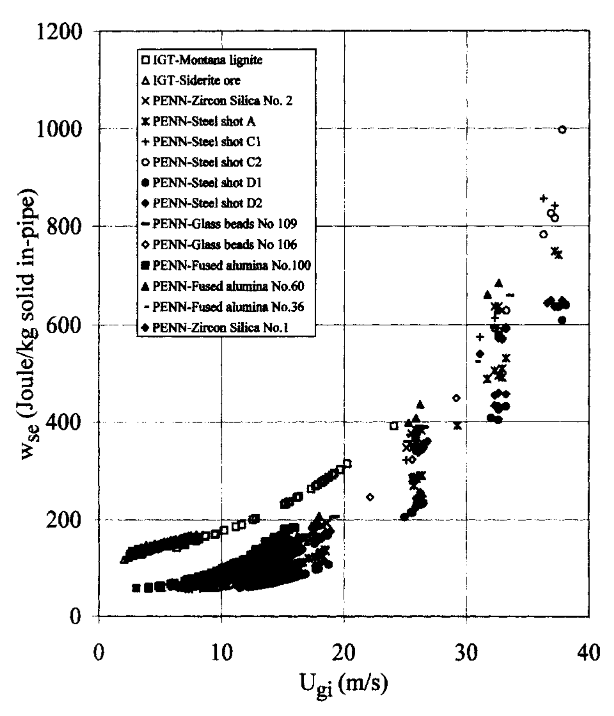

Figure 3. Solid effective work vs. inlet superficial gas velocity for IGT and Penn. Univ. data.

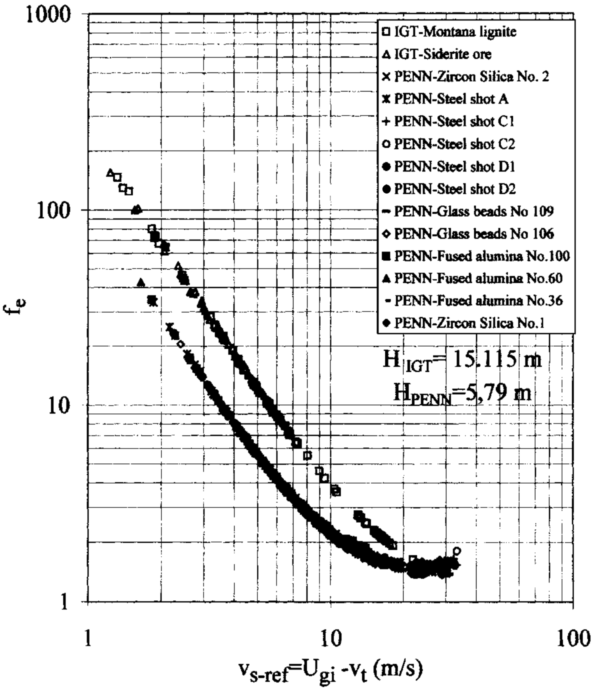

Figure 4. Energy factor vs. solid reference velocity for IGT and Penn. Univ. data.

This is true for the effective work (Eq. 17) and the solid effective work (Eq. 20) also.

The theoretical demonstration of the slip ratio constancy was exclusively derived for a 1-D steady two-phase flow, and it was only based on mass conservation and continuity (Collado and Muñoz, 1997), with neither relevant to the demon-

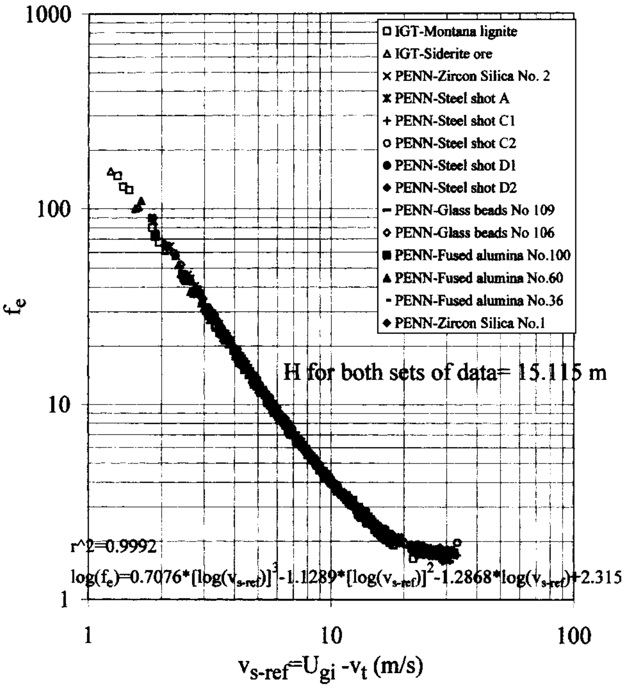

Figure 5. Energy factor vs. solid reference velocity for IGT and Penn. Univ. data (both with IGT height).

stration if the solids were accelerated or not (provided that we can assume a steady 1-D solid velocity field) nor if we treat with dense or dilute transports.

Thus, Eq. 3 would be theoretically valid just from the point of the lift line, where a net upward average velocity of the solid was established. Then, it would be verified for almost all the lift line, excluding the short section after the feeding point where the solid particles would be changing from horizontal direction to vertical one.

The physical need of the slip ratio constancy has been advanced at the introduction. The slip ratio act as a time scale factor between the phases which have different velocities. The key point is that we are treating simultaneously two different velocities in the same length (that of the control volume). Thus, having in mind velocity concept, it is completely impossible that the time scales of the two phases be the same

$$v_g = \frac{dz}{dt_g} \neq v_s = \frac{dz}{dt_s} \Rightarrow \frac{v_g}{v_s} = S = \frac{dt_s}{dt_g} \tag{25}$$

Equation 3 would merely propose that for a 1-D steady state this time scale factor between the phases should be a constant along the pipe. Unfortunately, the classical equations do not consider at all this physical fact.

On the other hand, the slip ratio constancy is also used to work out an approximate value of the outlet solid velocity, which is needed in Eqs. 19 and 20. We assume a "reasonable" value of the slip ratio for the line, which we have chosen equal to the quotient of the superficial inlet gas velocity and the solid reference velocity (Eq. 23). Calculating the outlet gas velocity, we can work out the outlet solid velocity with the help of this "reasonable" slip ratio.

From Eq. 3, it could be rightly argued that this "reasonable" slip value implies to assume a "fictitious" inlet vertical solid velocity equal to

$$v_{gi}/v_{si} \approx S \approx U_{gi}/v_{s-\mathrm{ref}} \Rightarrow v_{si} \approx U_{gi} - v_t \tag{26}$$

However, working with 1-D models, we will always have a discontinuity in Eq. 2 just at $z = 0$ in the systems (as the IGT and Penn tests) where the solids enter the line with zero vertical velocity.

For solving this problem, as we have already commented, Arastoopour and Gidaspow (1979a,b) suggest to assume a "reasonable" inlet solid void fraction to fit the pressure drop data. Here, the proposed solution to this unavoidable discontinuity is based on the constancy of the slip ratio along the lift line (Eq. 3), and the assumption of a "reasonable" value of the slip (Eq. 23). The clear advantage over the Arastoopour and Gidaspow (1979a,b) models other than the accuracy of the correlation found, is that the calculation procedure is clear and straightforward and not necessary to tune the assumption for fitting the pressure drop data.

Furthermore, this "fictitious" inlet vertical solid velocity is not directly included neither in the effective work, nor the solid effective work (Eqs. 19 and 20). Thus, Eq. 23 does not at all mean that the particles are already wholly accelerated at the inlet and that there is no acceleration region in the duct.

Indeed, in the IGT lift line, the first pressure drop tap was just below the solid feeding port, whereas in the Pennsylvania University line, it was just at the outlet of a short bend next to the solid particles entrance. Thus, for both set of tests, a zero vertical component of the particles velocity ($v_{si} = 0$) was assigned in Eqs. 19 and 20. In conclusion, at the difference of the recommended empirical correlations, this implies that the reported pressure drop data and the new correlations proposed (see Figures 4 and 5) definitely include the important term (Fan and Zhu, 1998) of the acceleration of the solids after the entrance.

On the other hand, the data of Knowlton and Bachovchin (1976) lie mostly in the "fast fluidization" region in which circulating fluidized beds operate and reflux occurs. Nevertheless, even though one of the main assumptions of the derivation of Eq. 3 is that there is no refluxing in the pipe (Collado and Muñoz, 1997), the correlations work quite well (see Figures 4 and 5).

Mechanical Losses for 1-D Steady Gas-Solid Flow

Finally, we will briefly outline in this paragraph the main mechanical losses in a gas-solid system following the new point of view. These losses would correspond with the mechanical nonequilibrium internal to the global gas-solid system in particular, the global friction of the mixture with the pipe wall (or the velocity gradient near the wall), and the internal friction in this 1-D system due to the difference of velocities between the phases (the slip). It is well known from classical thermodynamics (Hall, 1956; Zucker, 1977) that the comparison of the pressure-energy equation with the momentum equation should allow to identify the mechanical irreversibilities in a fluid system.

We will first derive a new 1-D steady-state global momentum balance for a vertical gas-solid flow following the new presented procedure.

New momentum balance for 1-D steady gas-solid flow

The vertical, 1-D momentum equation states (Hunsaker and Rightmire, 1947) that the resultant force Fz (pressure, wall friction and gravity) on the global gas-solid system, momentarily occupying a fixed differential control volume of height dz, equals the net rate of outflow of z momentum through the control volume (in Newton):

$$A_c dP + \tau_w \pi D dz + g \rho_m A_c dz + d\{(\rho_m m_m) v_g A_c\} = 0 \tag{27}$$

where τ_w is the gas-solid average shear stress at the pipe wall. Again as in the case of the energy balance, the gas velocity is chosen as representative mixture velocity. By coherence with the new modeling procedure, we define a mixture momentum m_m (per unit mass of mixture) as

$$m_m = (\rho_s(1-\epsilon)/\rho_m) v_s + (\rho_g \epsilon/\rho_m) v_g \tag{28}$$

Substituting Eq. 28 in Eq. 27, we yield

$$A_c dP + \tau_w \pi D dz + g \rho_m A_c dz + d\{[\rho_s(1-\epsilon) v_s + \rho_g \epsilon v_g] v_g A_c\} = 0 \tag{29}$$

Dividing Eq. 29 by $A_c dz$ and reordering terms, we have the differential mixture momemtum balance (in Newton/m^3)

$$\rho_s(1-\epsilon)v_s\frac{dv_g}{dz}+\rho_g\epsilon v_g\frac{dv_g}{dz}+g\rho_m=-\frac{dP}{dz}-\frac{4\tau_w}{D} \quad (30)$$

For the sake of comparison with classical 1-D momentum balances (Arastoopour and Gidaspow, 1979a,b; Aratoopour et al., 1982), we again arrange the terms in Eq. 30 stating

$$\rho_s(1-\epsilon)v_s\frac{dv_s}{dz}+\rho_g\epsilon v_g\frac{dv_g}{dz}+g\rho_m+\rho_s(1-\epsilon)v_s\left(\frac{v_g}{v_s}-1\right)\frac{dv_s}{dz}=-\frac{dP}{dz}-\frac{4\tau_w}{D} \quad (31)$$

Note that we have used in the above equation the fact that the slip ratio is a constant through the pipe. Also, note that the only difference with the standard expressions is the last term on the lefthand side of Eq. 31, which accounts for the pressure drop due to the nonequilibrium of velocities between the phases (slip). Coherently, if the slip was equal to one (equilibrium of velocities between the phases), this term would be zero.

New mechanical energy balance for 1-D gas-solid flow

From the momentum equation (Eq. 31), we might obtain a mechanical energy balance: first, we multiply Eq. 31 by the product of the cross-sectional area and the *dz* to identify the gas and the solids mass-flow rates (Eqs. 1–2). Also, we group the gas and solids terms separately

$$-\left\{\rho_g\epsilon A_c(dP/\rho_g)+\rho_g\epsilon A_c gdz+\dot{m}_g dv_g\right\}=\rho_s(1-\epsilon)A_c(dP/\rho_s)+\rho_s(1-\epsilon)A_c gdz+\dot{m}_s Sdv_s+\tau_w\pi Ddz \quad (32)$$

Now we multiply this momemtum equation (Eq. 32) by the mixture velocity (in this case the gas velocity)

$$-\dot{m}_g\left\{(dP/\rho_g)+gdz+v_g dv_g\right\}=\dot{m}_s\cdot S\cdot\left\{dP/\rho_s+gdz+S\cdot v_s\cdot dv_s\right\}+\tau_w\pi Dv_g dz \quad (33)$$

Finally, dividing Eq. 33 by the gas mass-flow rate, a mechanical energy balance of the gas-solid flow in Joule per unit mass of gas is obtained

$$-\left\{(dP/\rho_g)+gdz+v_g dv_g\right\}=\eta\cdot S\cdot\left\{dP/\rho_s+gdz+S\cdot v_s\cdot dv_s\right\}+(4\tau_w dz)/(\rho_g\epsilon D) \quad (34)$$

Identification of the dissipation terms in gas-solid flow

The differential expression of the pressure-energy equation (Eq. 16) will be

$$-\left\{(dP/\rho_g)+gdz+v_g dv_g\right\}=\eta\cdot S\cdot\left\{dP/\rho_s+gdz+v_s\cdot dv_s\right\}+\delta w_{\text{lost}} \quad (35)$$

Then, substracting Eq. 35, the differential pressure-energy equation from Eq. 34, the differential mechanical energy balance, we obtain a clear identification of the lost work terms (per unit mass of gas)

$$\delta w_{\text{lost}}=\eta\cdot S\cdot(S-1)\cdot v_s\cdot dv_s+\frac{4\tau_w}{\rho_g\epsilon D}dz \quad (36)$$

In Eq. 36, the first term of the lost work represents the slip dissipation due to the difference of velocities between the phases, which is dependent on the load ratio, the solid velocity, and the slip ratio. The second term accounts for the global wall friction of the mixture which depends on the value of the average wall friction, and also on the length and the diameter of the pipe.

Conclusions

Based on thermodynamic fundamentals and a new strict treatment of the slip between the phases, a new pressure-energy equation for a 1-D steady gas-solid flow is presented. From this equation, a new nondimensional energy factor, which includes the total pressure drop, has been obtained. Such a factor has been checked against high-pressure data from the IGT lift line of 15.115 m height, and the atmospheric-pressure data from the Pennsylvania University lift line of 5.79 m height.

A total of 923 tests were performed where the pressure drop was taken just from the inlet, which includes dilute and dense phase transports, several pipe diameters, broad and narrow particle-size distributions, and very different materials.

As a major result, for both sets of pressure drop data, a quite strong global correlation of the energy factor with a reference solid velocity (clearly defined as the superficial inlet gas velocity minus the solid terminal velocity) has been found. Not to mention the large set of data quite well correlated, the model presents some clear advantages over the classical hydrodynamic models because we do not need to fit an unknown "reasonable" inlet solid voidage (Arastoopour and Gidaspow, 1979a,b) and the problems due to broad particle-size distributions (Arastoopour et al., 1982) can been readily overcome with the new approach. In reference to the empirical correlations, in addition to the problem of the poor accuracy, notice that the recommended ones were written for data taken in the fully developed region. Thus, they do not take into account the pressure drop over the acceleration region of the solids after the entrance, which is often significant or even dominant (Fan and Zhu, 1998); whereas the model proposed here does include successfully the pressure drop due to inlet solids acceleration. Finally, following the new approach, a new momentum balance was derived. From the comparison of the momentum and pressure-energy equations, the mechanical losses due to global wall friction and slip between the phases were clearly identified.

Notation

A_c = cross-sectional area of the line, m^2
c = solid heat capacity, J/kg·K
e = specific energy, J/kg
ΔP = total pressure drop in the lift line, N·m^{-2}
f_e = nondimensional energy factor (Eq. 21)

g = gravitational constant, 9.81 $m \cdot s^{-2}$
h = gas specific enthalpy, J/kg
H = height of the lift line, m
$\dot{m}$ = mass-flow rate, $kg \cdot s^{-1}$
m_m = specific mixture momentum, $m \cdot s^{-1}$
P = pressure, $N \cdot m^{-2}$
q = heat exchanged per unit mass of gas, J/kg gas
$\dot{Q}_m$ = heat exchanged per unit time by the mixture, W
s = specific entropy, J/kg·K
S = slip ratio (see Eq. 3)
T = temperature, K
u = specific internal energy, J/kg
U_{gi} = superficial inlet gas velocity, $m \cdot s^{-1}$
v = velocity, $m \cdot s^{-1}$
v_t = solid terminal velocity, ms^{-1}
v_{s-ref} = solid reference velocity, $m \cdot s^{-1}$ (Eq. 22)
w_e = effective work, J/kg gas (see Eq. 19)
w_{lost} = lost work, J/kg gas (see Eq. 15)
w_{se} = solid effective work, J/kg solid-in-line (Eq. 20)
$\dot{W}_m$ = work exchanged per unit time by the mixture, W
z = vertical coordinate, m

Greek letters

ϵ = cross-sectional average gas voidage
η = solid-gas mass flow rate ratio
ρ = density, $kg \cdot m^{-3}$
τ_w = gas-solid average shear stress at the pipe wall, $N \cdot m^{-2}$

Subscripts

g = gas
i = inlet
m = gas-solid mixture
o = outlet
s = solid

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